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by parmenides51

1 Let x, y and z be rational numbers satisfying

$$x^3 + 3y^3 + 9z^3 - 9xyz = 0.$$

Prove that $x = y = z = 0$.

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2 Prove that $f(2) \geq 3^n$ where the polynomial $f(x) = x_n + a_1 x_{n-1} + ... + a_{n-1} x + 1$ has non-negative coefficients and n real roots.

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3 Given are $n + 1$ points $P_1, P_2, ..., P_n$ and Q in the plane, no three collinear. For any two different points P_i and P_j , there is a point P_k such that the point Q lies inside the triangle $P_i P_j P_k$. Prove that n is an odd number.
