Austria Beginners' Competition 2005

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by parmenides51

1 Show that there are no positive integers a und b such that $4a(a+1) = b(b+3)$

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2 Determine the number of integer pairs (x, y) such that $(|x| - 2)^2 + (|y| - 2)^2 < 5$.

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3 Determine all triples (x, y, z) of real numbers that satisfy all of the following three equations:

$$\begin{cases} \lfloor x \rfloor + \{y\} = z \\ \lfloor y \rfloor + \{z\} = x \\ \lfloor z \rfloor + \{x\} = y \end{cases}$$

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4 We are given the triangle ABC with an area of 2000. Let P, Q, R be the midpoints of the sidess BC, AC, AB. Let U, V, W be the midpoints of the sides QR, PR, PQ. The lengths of the line segments AU, BV, CW are x, y, z. Show that there exists a triangle with side lengths x, y and z and caluclate it's area.
