

Austria Beginners' Competition 2006

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by parmenides51

1 Do integers a, b exist such that $a^{2006} + b^{2006} + 1$ is divisible by 2006^2 ?

2 For which real numbers a is the set of all solutions of the inequality

$$(x^2 + ax + 4)(x^2 - 5x + 6) < 0$$

an interval?

3 Let n be an even positive integer. We consider rectangles with integer side lengths k and $k + 1$, where k is greater than $\frac{n}{2}$ and at most equal to n . Show that for all even positive integers n the sum of the areas of these rectangles equals

$$\frac{n(n+2)(7n+4)}{24}.$$

4 Show that if a triangle has two excircles of the same size, then the triangle is isosceles.

(Note: The excircle ABC to the side a touches the extensions of the sides AB and AC and the side BC .)
