

**Austria Beginners' Competition 2007**

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by parmenides51

- 1 Prove that the number  $9^n + 8^n + 7^n + 6^n - 4^n - 3^n - 2^n - 1^n$  is divisible by 10 for all non-negative  $n$ .
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- 2 Find all real solutions to the equation

$$[x]^2 + [x] = x^2 - \frac{1}{4}.$$

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- 3 For real numbers  $x \geq 0$  and  $y \geq 0$ , write  $A = \frac{x+y}{2}$  for the arithmetic mean and  $G = \sqrt{xy}$  for the geometric mean of  $x$  and  $y$ . Furthermore, let  $W = \frac{\sqrt{x} + \sqrt{y}}{2}$  be the arithmetic mean of  $\sqrt{x}$  and  $\sqrt{y}$ . Prove that

$$G \leq W^2 \leq A.$$

Determine all  $x$  and  $y$  such that  $G = W^2 = A$

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- 4 Consider a parallelogram  $ABCD$  such that the midpoint  $M$  of the side  $CD$  lies on the angle bisector of  $\angle BAD$ . Show that  $\angle AMB$  is a right angle.
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