## AoPS Community

## Austria Beginners' Competition 2008

www.artofproblemsolving.com/community/c3172423
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1 Determine all positive integers $n$ such that $\frac{2^{n}}{n^{2}}$ is an integer.
2 Determine all real numbers $x$ satisfying

$$
x\lfloor x\lfloor x\rfloor\rfloor=\sqrt{2} .
$$

3 Prove the inequality

$$
\frac{a+b}{a^{2}-a b+b^{2}} \leq \frac{4}{|a+b|}
$$

for all real numbers $a$ and $b$ with $a+b \neq 0$. When does equality hold?
4 Let $A B C$ be an acute-angled triangle with the property that the bisector of $\angle B A C$, the altitude through $B$ and the perpendicular bisector of $A B$ intersect in one point. Determine the angle $\alpha=\angle B A C$.

