

Kurschak Competition 1974

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- 1 A library has one exit and one entrance and a blackboard at each. Only one person enters or leaves at a time. As he does so he records the number of people found/remaining in the library on the blackboard. Prove that at the end of the day exactly the same numbers will be found on the two blackboards (possibly in a different order).
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- 2 S_n is a square side $\frac{1}{n}$. Find the smallest k such that the squares S_1, S_2, S_3, \dots can be put into a square side k without overlapping.
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- 3 Let

$$p_k(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + \frac{(-x)^{2k}}{(2k)!}$$

Show that it is non-negative for all real x and all positive integers k .
