Kurschak Competition 1976

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by parmenides51

1 $ABCD$ is a parallelogram. P is a point outside the parallelogram such that angles $\angle PAB$ and $\angle PCB$ have the same value but opposite orientation. Show that $\angle APB = \angle DPC$.

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2 A lottery ticket is a choice of 5 distinct numbers from $1, 2, 3, ..., 90$. Suppose that 55 distinct lottery tickets are such that any two of them have a common number. Prove that one can find four numbers such that every ticket contains at least one of the four.

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3 Prove that if the quadratic $x^2 + ax + b$ is always positive (for all real x) then it can be written as the quotient of two polynomials whose coefficients are all positive.
