

Kurschak Competition 1978

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- 1 a and b are rationals. Show that if $ax^2 + by^2 = 1$ has a rational solution (in x and y), then it must have infinitely many.

- 2 The vertices of a convex n -gon are colored so that adjacent vertices have different colors. Prove that if n is odd, then the polygon can be divided into triangles with non-intersecting diagonals such that no diagonal has its endpoints the same color.

- 3 A triangle has inradius r and circumradius R . Its longest altitude has length H . Show that if the triangle does not have an obtuse angle, then $H \geq r + R$. When does equality hold?
