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by parmenides51, rcorreaa

– Day 1

1 A single player game has the following rules: initially, there are 10 piles of stones with $1, 2, ..., 10$ stones, respectively. A movement consists on making one of the following operations:

i) to choose 2 piles, both of them with at least 2 stones, combine them and then add 2 stones to the new pile;

ii) to choose a pile with at least 4 stones, remove 2 stones from it, and then split it into two piles with amount of piles to be chosen by the player.

The game continues until is not possible to make an operation. Show that the number of piles with one stone in the end of the game is always the same, no matter how the movements are made.

2 Let ABC be an acute triangle, with $AB < AC$. Let K be the midpoint of the arch BC that does not contain A and let P be the midpoint of BC. Let I_B, I_C be the B-excenter and C-excenter of ABC, respectively. Let Q be the reflection of K with respect to A. Prove that the points P, Q, I_B, I_C are concyclic.

3 Let $\{a_n\}_{n=0}^{\infty}$ be a sequence of integers numbers. Let $\Delta^1 a_n = a_{n+1} - a_n$ for a non-negative integer n. Define $\Delta^M a_n = \Delta^{M-1} a_{n+1} - \Delta^{M-1} a_n$. A sequence is *patriota* if there are positive integers k, l such that $a_{n+k} = \Delta^M a_{n+l}$ for all non-negative integers n. Determine, with proof, whether exists a sequence that the last value of M for which the sequence is *patriota* is 2022.

– Day 2

4 Initially, a natural number n is written on the blackboard. Then, at each minute, *Neymar* chooses a divisor $d > 1$ of n, erases n, and writes $n + d$. If the initial number on the board is 2022, what is the largest composite number that *Neymar* will never be able to write on the blackboard?

5 Let n be a positive integer number. Define $S(n)$ to be the least positive integer such that $S(n) \equiv n \pmod 2$, $S(n) \geq n$, and such that there are **not** positive integers numbers $k, x_1, x_2, ..., x_k$ such that $n = x_1 + x_2 + ... + x_k$ and $S(n) = x_1^2 + x_2^2 + ... + x_k^2$. Prove that there exists a real constant $c > 0$ and a positive integer n_0 such that, for all $n \geq n_0$, $S(n) \geq cn^{\frac{3}{2}}$.

6 Some cells of a 10×10 are colored blue. A set of six cells is called *gremista* when the cells are the intersection of three rows and two columns, or two rows and three columns, and are painted blue. Determine the greatest value of n for which it is possible to color n chessboard cells blue such that there is not a *gremista* set.