



Centroamerican and Caribbean Math Olympiad 2022

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by AlanLG

– Day 1

1 There is a pile with 2022 rocks. Ana y Beto play by turns to the following game, starting with Ana: in each turn, if there are n rocks in the pile, the player can remove $S(n)$ rocks or $n - S(n)$ rocks, where $S(n)$ is the sum of the the digits of n . The person who removes the last rock wins. Determine which of the two players has a winning strategy and describe it.

2 Ana, Beto, Carlos, Diana, Elena and Fabian are in a circle, located in that order. Ana, Beto, Carlos, Diana, Elena and Fabian each have a piece of paper, where are written the real numbers a, b, c, d, e, f respectively.

At the end of each minute, all the people simultaneously replace the number on their paper by the sum of three numbers; the number that was at the beginning of the minute on his paper and on the papers of his two neighbors. At the end of the minute 2022, 2022 replacements have been made and each person have in his paper it's initial number. Find all the posible values of $abc + def$.

Note: [i]f at the beginning of the minute N Ana, Beto, Carlos have the numbers x, y, z , respectively, then at the end of the minute N , Beto is going to have the number $x + y + z[i]$.

3 Let ABC an acutangle triangle with orthocenter H and circumcenter O . Let D the intersection of AO and BH . Let P be the point on AB such that $PH = PD$. Prove that the points B, D, O and P lie on a circle.

– Day 2

4 Let $A_1A_2A_3A_4$ be a rectangle and let S_1, S_2, S_3, S_4 four circumferences inside of the rectangle such that S_k and S_{k+1} are tangent to each other and tangent to the side A_kA_{k+1} for $k = 1, 2, 3, 4$, where $A_5 = A_1$ and $S_5 = S_1$. Prove that $A_1A_2A_3A_4$ is a square.

5 Esteban the alchemist have 8088 copper pieces, 6066 bronze pieces, 4044 silver pieces and 2022 gold pieces. He can take two pieces of different metals and use a magic hammer to turn them into two pieces of different metals that he take and different each other. Find the largest number of gold pieces that Esteban can obtain after using the magic hammer a finite number of times.

Note: If Esteban takes a copper and bronze pieces, then he turn them into a silver and a gold pieces.

6 A positive integer n is *inverosimil* if there exists n integers not necessarily distinct such that

the sum and the product of this integers are equal to n . How many positive integers less than or equal to 2022 are *inverosimils*?
