Dutch BxMO Team Selection Test 2022

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by parmenides51

1 Find all functions $f : Z_{>0} \to Z_{>0}$ for which $f(n)|f(m) - n$ if and only if $n|m$ for all natural numbers m and n.

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2 Let ABC be an acute triangle, and let D be the foot of the altitude from A. The circle with centre A passing through D intersects the circumcircle of triangle ABC in X and Y , in such a way that the order of the points on this circumcircle is: A, X, B, C, Y . Show that $\angle BXD = \angle CYD$.

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3 Find all pairs (p, q) of prime numbers such that

$$p(p^2 - p - 1) = q(2q + 3).$$

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5 At a fish market there are 10 stalls, each selling the same 10 kinds of fish. Each fish was caught in either the North Sea or the Mediterranean Sea, and each stall has, for each kind of fish, only fish of one origin. A number, say k, of customers buy exactly one fish from each stall, in such a way that they obtain exactly one of each kind of fish. oreover, for each pair of customers, there is a kind of fish of which the customers have fish of different origin. Consider all possible ways to supply the stalls according to the rules above. What is the largest possible value of k?
