Spain Mathematical Olympiad 1966

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–	Day 1
1	To a manufacturer of three products whose unit prices are 50, 70, and 65 pta, a retailer asks him for 100 units, remitting him 6850 pta as payment, on the condition that you send as many of the higher-priced product as possible and the rest of the other two. How many of each product should he send to serve the request?
2	A three-digit number is written xyz in the base 7 system and zyx in the base 9 system . What is the number?
3	Given a regular pentagon, consider the convex pentagon limited by its diagonals. You are asked to calculate: a) The similarity relation between the two convex pentagons. b) The relationship of their areas. c) The ratio of the homothety that transforms the first into the second.
4	You want to hang a weight P so that it is 7 m below a ceiling. To do this, it is suspended by means of a vertical cable attached to the midpoint M of a chain hung by its ends from two points on the ceiling A and B distant from each other 4 m. The price of the cable PM is p pta/m and that of the chain AMB is q pta/m. It is requested: a) Determine the lengths of the cable and the chain to obtain the lowest price cost of installation. b) Discuss the solution for the different values of the relation p/q of both prices. (It is assumed that the weight is large enough to be considered rectile lines the chain segments AM and MB).
–	Day 2
5	The length of the hypotenuse BC of a right triangle ABC is a, and on it the points M and N are taken such that $BM = NC = k$, with $k < a/2$. Assuming that (only) the data a and k are known, calculate: a) The value of the sum of the squares of the lengths AM and AN. b) The ratio of the areas of triangles ABC and AMN. c) The area enclosed by the circle that passes through the points A, M', N' , where M' is the orthogonal projection of M onto AC and N' that of N onto AB.
6	They tell us that a married couple has 5 children. Calculate the probability that among them there are at least two men and at least one woman. Probability of being born male is considered $1/2$.

7 Determine a geometric progression of seven terms, knowing the sum, 7, of the first three, and the sum, 112, of the last three.

8 Determine the values of a, b, c, so that the graphical representation of the function

$$y = ax^3 + bx^2 + cx$$

has an inflection point at the point of abscissa $x = 3$, with tangent at the point of equation $x - 4y + 1 = 0$. Then draw the corresponding graph.