Spain Mathematical Olympiad 1967

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–	Day 1
1	It is known that the real function $f(t)$ is monotonic increasing in the interval $-8 \leq t \leq 8$, but nothing is known about what happens outside of it. In what range of values of x, can it be ensured that the function $y = f(2x - x^2)$ is monotonic increasing?
2	Determine the poles of the inversions that transform four collienar points A, B, C, D, aligned in this order, at four points A', B', C' , D' that are vertices of a rectangle, and such that A' and C' are opposite vertices.
3	A traffic light installed at a main junction of a road, in which you circulate in both directions, it remains red for 30 s and green for another 30 s, alternately. You want to install another traffic light on the same road, for a secondary crossing, located 400 m away from the first, which works with the same period of 1 min duration. It is wanted that the cars that circulate at 60 km/h on the road in any of the two senses and that they do not have to stop if there was only the traffic light of the main intersection. They also don't have to stop after installing the secondary crossover. How many seconds can red be on at the secondary traffic light? Note: It is suggested to reason on a Cartesian representation of the march of the cars, taking an axis of distances and another of times.
4	There is a bottle with a flat and circular bottom, closed and partially filled of wine, so that its level does not exceed the cylindrical part. Discuss in which cases the capacity of the bottle can be calculated without opening it, having only one double graduated decimeter; and if possible, describe how it would be calculated. (Problem of the Italian *Gara Mathematica*).
–	Day 2
5	Let γ be a semicircle with diameter AB . A creek is built with origin in A , which has its vertices alternately in the diameter AB and in the semicircle γ , so that its sides make equal angles α with the diameter (but alternately in either direction). It is requested: a) Values of the angle α for the ravine to pass through the other end B of the diameter. b) The total length of the ravine, in the case that it ends in B , as a function of the length d of the diameter and of the angle α . https://cdn.artofproblemsolving.com/attachments/3/c/54c71cdf1bf8fbb3bdfa38f4b14a1dc961c5f.png

6 An equilateral triangle ABC with center O and radius $OA = R$ is given, and consider the seven regions that the lines of the sides determine on the plane. It is asked to draw and describe the region of the plane transformed from the two shaded regions in the attached figure, by the inversion of center O and power R^2.
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png

7 On a road a caravan of cars circulates, all at the same speed, maintaining the minimum separation between one and the other indicated by the Code of Circulation. This separation is, in meters, $\frac{u^2}{100}$, where u is the speed expressed in km/h. Assuming that the length of each car is 2.89 m, calculate the speed at which they must circulate so that the capacity of traffic is maximum, that is, so that in a fixed time the maximum number pass of vehicles at a point on the road.

8 To obtain the value of a polynomial of degree n, whose coefficients are

$$a_0, a_1, ..., a_n$$

(starting with the term of highest degree), when the variable x is given the value b, the process indicated in the attached flowchart can be applied, which develops the actions required to apply Ruffini's rule. It is requested to build another flowchart analogous that allows to express the calculation of the value of the derivative of the given polynomial, also for $x = b$.
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