Spain Mathematical Olympiad 1969

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–	Day 1
1	Find the locus of the centers of the inversions that transform two points A, B of a given circle γ , at diametrically opposite points of the inverse circles of γ .
2	Find the locus of the affix M, of the complex number z, so that it is aligned with the affixes of i and iz .
3	A bag contains plastic cubes of the same size, whose faces have been painted in colors: white, red, yellow, green, blue and violet (without repeating a color on two faces of the same cube). How many of these cubes can there be distinguishable to each other?
4	A circle of radius R is divided into 8 equal parts. The points of division are denoted successively by A, B, C, D, E, F, G and H. Find the area of the square formed by drawing the chords AF , BE, CH and DG.
–	Day 2
5	Show that a convex polygon with more than four sides cannot be decomposed into two others, both similar to the first (directly or inversely), by means of a single rectilinear cut. Reasonably specify which are the quadrilaterals and triangles that admit a decomposition of this type.
6	Given a polynomial of real coefficients P(x) , can it be affirmed that for any real value of x is true of one of the following inequalities: $$P(x) \leq P(x)^2;\;\; P(x) < 1 + P(x)^2;\;\; P(x) \leq \frac{1}{2} + \frac{1}{2}P(x)^2.$$ Find a simple general procedure (among the many existing ones) that allows, provided we are given two polynomials $P(x)$ and $Q(x)$, find another $M(x)$ such that for every value of x, at the same time $-M(x) < P(x) < M(x)$ and $-M(x) < Q(x) < M(x)$.
7	A convex polygon $A_1A_2...A_n$ of n sides and inscribed in a circle, has its sides that satisfy the inequalities $$A_nA_1 > A_1A_2 > A_2A_3 > ... > A_{n-1}A_n$$ Show that its interior angles satisfy the inequalities $$\angle A_1 < \angle A_2 < \angle A_3 < ... < \angle A_{n-1}, \angle A_{n-1} > \angle A_n > \angle A_1.$$

8 The house SEAT recommends to the users, for the correct conservation of the wheels, periodic replacements of the same in the form $R \to 3 \to 2 \to 1 \to 4 \to R$, according to the numbering of the figure. Calling G to this change of wheels, $G^2 = GG$ to making this change twice, and so on for the other powers of G,
a) Show that the set of these powers forms a group, and study it.
b) Each puncture of one of the wheels is also equivalent to a substitution in which said wheel is replaced by the spare one (R) and, once repaired, it comes to occupy the place of this obtained G as a product of prick transformations. Do they form a group?
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