

Spain Mathematical Olympiad 1970

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– Day 1

1 A cylindrical container of revolution is partially filled with a liquid whose density we ignore. Placing it with the axis inclined 30° with respect to the vertical, we observe that when removing liquid so that the level falls 1 cm, the weight of the contents decreases 40 g. How much will the weight of that content decrease for each centimeter that lower the level if the axis makes an angle of 45° with the vertical? It is supposed that the horizontal surface of the liquid does not touch any of the bases of the container.

2 A plant grows in the way we describe below. has a trunk which forks into two branches; each branch of the plant can, in turn, branch off into other two branches, or end in a bud. We will call the *load* of a branch the total number of buds it bears, that is, the number of buds fed by the sap that passes by that branch; and we will call the *distance* of a bud the number of bifurcations that it sap has to go through to get from the trunk to that bud.

If n is the number of bifurcations that a certain plant of that type has, it is asks

a) the number of branches of the plant,

b) the number of buds,

c) show that the sum of the charges of all the branches is equal to the sum of the clearances of all buds.

Hint: You can proceed by induction, showing that if some results are correct for a given plant, they remain correct for the plant that is obtained substituting a bud in it for a pair of branches ending in individual buds.

3 An arbitrary triangle ABC is given and a point P lies on the side AB . It is requested to draw through P a line that divides the triangle into two figures of the same area.

4 Knowing that the polynomials

$$2x^5 - 13x^4 + 4x^3 + 61x^2 + 20x - 25$$

$$x^5 - 4x^4 - 13x^3 + 28x^2 + 85x + 50$$

have two common double roots, determine all their roots.

– Day 2

5 In the sixth-year exams of a Center, they pass Physics at least 70% of the students, Mathematics at least 75%; Philosophy at least, the 90% and the Language at least, 85%. How many students, at least, pass these four subjects?

6 Given a circle γ and two points A and B in its plane. By B passes a variable secant that intersects γ at two points M and N . Determine the locus of the centers of the circles circumscribed to the triangle AMN .

7 Calculate the values of the cosines of the angles x that satisfy the next equation:

$$\sin^2 x - 2 \cos^2 x + \frac{1}{2} \sin 2x = 0.$$

8 There is a point M inside a circle, at a distance $OM = d$ of the center O . Two chords AB and CD are traced through M that form a right angle. Join A with C and B with D . Determine the cosine of the angle that must form the chord AB with OM so that the sum of the areas of the triangles AMC and BMD be minimal.
