**Spain Mathematical Olympiad 1972**

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by parmenides51

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| – | Day 1 |
| 1 | Let $K$ be a ring with unit and $M$ the set of $2 \times 2$ matrices constituted with elements of $K$. An addition and a multiplication are defined in $M$ in the usual way between arrays. It is requested to:<br>a) Check that $M$ is a ring with unit and not commutative with respect to the laws of defined composition.<br>b) Check that if $K$ is a commutative field, the elements of$M$ that have inverse they are characterized by the condition $ad - bc \neq 0$.<br>c) Prove that the subset of $M$ formed by the elements that have inverse is a multiplicative group. |
| 2 | A point moves on the sides of the triangle $ABC$, defined by the vertices $A(-1.8, 0)$, $B(3.2, 0)$, $C(0, 2.4)$ . Determine the positions of said point, in which the sum of their distance to the three vertices is absolute maximum or minimum.<br>https://cdn.artofproblemsolving.com/attachments/2/5/9e5bb48cbeefaa5f4c069532bf5605b9c1f5e<br>png |
| 3 | Given a regular hexagonal prism. Find a polygonal line that, starting from a vertex of the base, runs through all the lateral faces and ends at the vertex of the face top, located on the same edge as the starting vertex, and has a minimum length. |
| 4 | The following sets of points are considered in the plane: $A = \{$ affixes of complexes $z$ such that arg $(z - (2 + 3i)) = \pi/4\}$, $B = \{$ affixes of complexes $z$ such that mod $(z - (2 + i) < 2\}$.<br>Determine the orthogonal projection on the $X$ axis of $A \cap B$. |
| – | Day 2 |
| 5 | Given two parallel lines $r$ and $r'$ and a point $P$ on the plane that contains them and that is not on them, determine an equilateral triangle whose vertex is point $P$ , and the other two, one on each of the two lines.<br>https://cdn.artofproblemsolving.com/attachments/9/3/1d475eb3e9a8a48f4a85a2a311e1bda978e74<br>png |
| 6 | Given three circumferences of radii $r$ , $r'$ and $r''$ , each tangent externally to the other two, calculate the radius of the circle inscribed in the triangle whose vertices are their three centers. |

 

**7** Prove that for every positive integer $n$, the number

$$A_n = 5^n + 2 \cdot 3^{n-1} + 1$$

is a multiple of $8$.

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**8** We know that $R^3 = \{(x_1, x_2, x_3) | x_i \in R, i = 1, 2, 3\}$ is a vector space regarding the laws of composition $(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$, $\lambda(x_1, x_2, x_3) = (\lambda x_1, \lambda x_2, \lambda x_3)$, $\lambda \in R$.
We consider the following subset of $R^3 : L = \{(x_1, x2, x_3) \in R^3 | x_1 + x_2 + x_3 = 0\}$.
a) Prove that $L$ is a vector subspace of $R^3$ .
b) In $R^3$ the following relation is defined $\overline{x} R \overline{y} \Leftrightarrow \overline{x} - \overline{y} \in L, \overline{x}, \overline{y} \in R^3$.
Prove that it is an equivalence relation.
c) Find two vectors of $R^3$ that belong to the same class as the vector $(-1, 3, 2)$.

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