German National Olympiad 2007

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by sqrtX, petros r

–	Day 1
1	Determine all real numbers x such that for all positive integers n the inequality $(1+x)^n \leq 1+(2^n-1)x$ is true.
2	Let A be the set of odd integers $\leq 2n-1$. For a positive integer m, let $B=\{a+m \,\vert\, a\in A\}$. Determine for which positive integers n there exists a positive integer m such that the product of all elements in A and B is a square.
3	We say that two triangles are oriented similarly if they are similar and have the same orientation. Prove that if $ALT, ARM, ORT,$ and ULM are four triangles which are oriented similarly, then A is the midpoint of the line segment OU.
–	Day 2
4	Find all triangles such that its angles form an arithmetic sequence and the corresponding sides form a geometric sequence.
5	Determine all finite sets M of real numbers such that M contains at least 2 numbers and any two elements of M belong to an arithmetic progression of elements of M with three terms.
6	For two real numbers a,b the equation: $x^4-ax^3+6x^2-bx+1=0$ has four solutions (not necessarily distinct). Prove that $a^2+b^2\geq 32$