German National Olympiad 2006

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–	Day 1
1	Find all $n \in \mathbb{Z}^+$, so that $$z_n = \underbrace{101\dots 101}_{2n+1 \text{ digits}}$$ is prime.
2	Five points are on the surface of of a sphere of radius 1. Let a_{min} denote the smallest distance (measured along a straight line in space) between any two of these points. What is the maximum value for a_{min}, taken over all arrangements of the five points?
3	For which positive integer n can you color the numbers 1,2...2n with n colors, such that every color is used twice and the numbers 1,2,3...n occur as difference of two numbers of the same color exatly once.
–	Day 2
4	Let D be a point inside a triangle ABC such that $\lvert AC\rvert - \lvert AD\rvert \geq 1$ and $\lvert BC\rvert - \lvert BD\rvert \geq 1$. Prove that for any point E on the segment AB, we have $\lvert EC\rvert - \lvert ED\rvert \geq 1$.
5	Let $x \neq 0$ be a real number satisfying $ax^2 + bx + c = 0$ with $a, b, c \in \mathbb{Z}$ obeying $\lvert a\rvert + \lvert b\rvert + \lvert c\rvert > 1$. Then prove $$\lvert x\rvert \geq \frac{1}{\lvert a\rvert + \lvert b\rvert + \lvert c\rvert - 1}.$$
6	Let a circle through B and C of a triangle ABC intersect AB and AC in Y and Z , respectively. Let P be the intersection of BZ and CY , and let X be the intersection of AP and BC . Let M be the point that is distinct from X and on the intersection of the circumcircle of the triangle XYZ with BC. Prove that M is the midpoint of BC