9th IGO

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by parmenides51, Msn05, a_507_bc, Tafi_ak

–	Elementary

1	Find the angles of the pentagon $ABCDE$ in the figure below.
	`https://cdn.artofproblemsolving.com/attachments/7/1/7cd8f03a4d968720d70a2f869509314b5a452`
	`jpeg`
	
	Proposed by Morteza Saghafian

2	An isosceles trapezoid $ABCD$ $(AB \parallel CD)$ is given. Points E and F lie on the sides BC and AD, and the points M and N lie on the segment EF such that $DF = BE$ and $FM = NE$. Let K and L be the foot of perpendicular lines from M and N to AB and CD, respectively. Prove that $EKFL$ is a parallelogram. *Proposed by Mahdi Etesamifard*
3	Let $ABCDE$ be a convex pentagon such that $AB = BC = CD$ and $\angle BDE = \angle EAC = 30^\circ$. Find the possible values of $\angle BEC$. *Proposed by Josef Tkadlec (Czech Republic)*
4	Let AD be the internal angle bisector of triangle ABC. The incircles of triangles ABC and ACD touch each other externally. Prove that $\angle ABC > 120^\circ$. (Recall that the incircle of a triangle is a circle inside the triangle that is tangent to its three sides.) *Proposed by Volodymyr Brayman (Ukraine)*
5	a) Do there exist four equilateral triangles in the plane such that each two have exactly one vertex in common, and every point in the plane lies on the boundary of at most two of them? b) Do there exist four squares in the plane such that each two have exactly one vertex in common, and every point in the plane lies on the boundary of at most two of them? (Note that in both parts, there is no assumption on the intersection of interior of polygons.) *Proposed by Hesam Rajabzadeh*
–	Intermediate

1	In the figure below we have $AX = BY$. Prove that $\angle XDA = \angle CDY$.
	`https://cdn.artofproblemsolving.com/attachments/4/6/509f2013e0da5573cc302ebd8f3f0b6b5b63c`
	`png`

Proposed by Iman Maghsoudi

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2 Two circles ω_1 and ω_2 with equal radius intersect at two points E and X. Arbitrary points C, D lie on ω_1, ω_2. Parallel lines to XC, XD from E intersect ω_2, ω_1 at A, B, respectively. Suppose that CD intersect ω_1, ω_2 again at P, Q, respectively. Prove that $ABPQ$ is cyclic.

Proposed by Ali Zamani

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3 Let O be the circumcenter of triangle ABC. Arbitrary points M and N lie on the sides AC and BC, respectively. Points P and Q lie in the same half-plane as point C with respect to the line MN, and satisfy $\triangle CMN \sim \triangle PAN \sim \triangle QMB$ (in this exact order). Prove that $OP = OQ$.

Proposed by Medeubek Kungozhin, Kazakhstan

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4 We call two simple polygons P, Q *compatible* if there exists a positive integer k such that each of P, Q can be partitioned into k congruent polygons similar to the other one. Prove that for every two even integers $m, n \geq 4$, there are two compatible polygons with m and n sides. (A simple polygon is a polygon that does not intersect itself.)

Proposed by Hesam Rajabzadeh

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5 Let $ABCD$ be a quadrilateral inscribed in a circle ω with center O. Let P be the intersection of two diagonals AC and BD. Let Q be a point lying on the segment OP. Let E and F be the orthogonal projections of Q on the lines AD and BC, respectively. The points M and N lie on the circumcircle of triangle QEF such that $QM \parallel AC$ and $QN \parallel BD$. Prove that the two lines ME and NF meet on the perpendicular bisector of segment CD.

Proposed by Tran Quang Hung, Vietnam

– Advanced Free

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1 Four points A, B, C and D lie on a circle ω such that $AB = BC = CD$. The tangent line to ω at point C intersects the tangent line to ω at A and the line AD at K and L. The circle ω and the circumcircle of triangle KLA intersect again at M. Prove that $MA = ML$.

Proposed by Mahdi Etesamifard

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2 We are given an acute triangle ABC with $AB \neq AC$. Let D be a point of BC such that DA is tangent to the circumcircle of ABC. Let E and F be the circumcenters of triangles ABD and ACD, respectively, and let M be the midpoints EF. Prove that the line tangent to the circumcircle of AMD through D is also tangent to the circumcircle of ABC.

Proposed by Patrik Bak, Slovakia

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3 In triangle ABC $(\angle A \neq 90^\circ)$, let O, H be the circumcenter and the foot of the altitude from A respectively. Suppose M, N are the midpoints of BC, AH respectively. Let D be the intersection of AO and BC and let H' be the reflection of H about M. Suppose that the circumcircle of $OH'D$ intersects the circumcircle of BOC at E. Prove that NO and AE are concurrent on the circumcircle of BOC.

Proposed by Mehran Talaei

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4 Let $ABCD$ be a trapezoid with $AB \parallel CD$. Its diagonals intersect at a point P. The line passing through P parallel to AB intersects AD and BC at Q and R, respectively. Exterior angle bisectors of angles DBA, DCA intersect at X. Let S be the foot of X onto BC. Prove that if quadrilaterals $ABPQ$, $CDQP$ are circumcribed, then $PR = PS$.

Proposed by Dominik Burek, Poland

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5 Let ABC be an acute triangle inscribed in a circle ω with center O. Points E, F lie on its side AC, AB, respectively, such that O lies on EF and $BCEF$ is cyclic. Let R, S be the intersections of EF with the shorter arcs AB, AC of ω, respectively. Suppose K, L are the reflection of R about C and the reflection of S about B, respectively. Suppose that points P and Q lie on the lines BS and RC, respectively, such that PK and QL are perpendicular to BC. Prove that the circle with center P and radius PK is tangent to the circumcircle of RCE if and only if the circle with center Q and radius QL is tangent to the circumcircle of BFS.

Proposed by Mehran Talaei
