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1 a, b, c, d are real numbers so that $a \geq b, c \geq d$,

$$|a| + 2|b| + 3|c| + 4|d| = 1.$$

Let $P = (a-b)(b-c)(c-d)$,find the maximum and minimum value of P.

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2 In acute triangle $\triangle ABC$, H is the orthocenter, BD,CE are altitudes. M is the midpoint of BC. P,Q are on segment BM,DE, respectively. R is on segment PQ such that $\frac{BP}{EQ} = \frac{CP}{DQ} = \frac{PR}{QR}$. Suppose L is the orthocenter of $\triangle AHR$, then prove: QM passes through the midpoint of RL.

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3 Does there exist an infinite set S consisted of positive integers,so that for any $x, y, z, w \in S, x < y, z < w$,if $(x, y) \neq (z, w)$,then $\gcd(xy + 2022, zw + 2022) = 1$?

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4 Given $r \in \mathbb{R}$. Alice and Bob plays the following game:
An equation with blank is written on the blackboard as below:

$$S = |\Box - \Box| + |\Box - \Box| + |\Box - \Box|$$

Every round, Alice choose a real number from $[0, 1]$ (not necessary to be different from the numbers chosen before) and Bob fill it in an empty box. After 6 rounds, every blank is filled and S is determined at the same time. If $S \geq r$ then Alice wins, otherwise Bob wins.
Find all r such that Alice can guarantee her victory.
