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1 $a_1, a_2, ..., a_9$ are nonnegative reals with sum 1. Define S and T as below:

$$S = \min\{a_1, a_2\} + 2\min\{a_2, a_3\} + ... + 9\min\{a_9, a_1\}$$

$$T = \max\{a_1, a_2\} + 2\max\{a_2, a_3\} + ... + 9\max\{a_9, a_1\}$$

When S reaches its maximum, find all possible values of T.

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2 A, B, C, D, E are points on a circle ω, satisfying $AB = BD$, $BC = CE$. AC meets BE at P. Q is on DE such that $BE//AQ$. Suppose $\odot(APQ)$ intersects ω again at T. A' is the reflection of A wrt BC. Prove that $A'BPT$ lies on the same circle.

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3 $S = \{1, 2, ..., N\}$. $A_1, A_2, A_3, A_4 \subseteq S$, each having cardinality 500. $\forall x, y \in S$, $\exists i \in \{1, 2, 3, 4\}$, $x, y \in A_i$. Determine the maximal value of N.

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4 $k > 2$ is an integer. $a_0, a_1, ...$ is an integer sequence such that $a_0 = 0$, $a_{n+1} = ka_n - a_{n-1}$. Prove that for any positive integer m, $(2m)!|a_1a_2...a_{3m}$.
