

**Turkey Junior National Olympiad 2022**

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- 1  $x, y, z$  are positive reals such that  $x \leq 1$ . Prove that

$$xy + y + 2z \geq 4\sqrt{xyz}$$

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- 2 In a school with 101 students, each student has at least one friend among the other students. Show that for every integer  $1 < n < 101$ , a group of  $n$  students can be selected from this school in such a way that each selected student has at least one friend among the other selected students.

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- 3 Let  $m, n, a, k$  be positive integers and  $k > 1$  such that the equality

$$5^m + 63n + 49 = a^k$$

holds. Find the minimum value of  $k$ .

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- 4 In parallelogram  $ABCD$ , on the arc  $BC$  of the circumcircle  $(ABC)$ , not containing the point  $A$ , we take a point  $P$  and on the  $[AC]$ , we take a point  $Q$  such that  $\angle PBC = \angle CDQ$ . Prove that  $(APQ)$  is tangent to  $AB$ .
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