Flanders Math Olympiad 2011

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1	Given are three numbers $a, b, c \in R - \{0\}$. The parabola with equation $y = ax^2 + bx + c$ lies above the line $y = cx$. Prove that the parabola with equation $y = cx^2 - bx + a$ lies above the line $y = cx - b$.
2	The area of the ground plane of a truncated cone K is four times as large as the surface of the top surface. A sphere B is circumscribed in K, that is to say that B touches both the top surface and the base and the sides. Calculate ratio volume B : Volume K.
3	There are 18 students in a class. Each student is asked two questions: how many other students have the same first name as you and how many other students have the same surname as you. The answers $0, 1, 2, ..., 7$ all occur. Prove that there are two students with the same first name and last name.
4	Given is a triangle ABC and points D and E, respectively on $]BC[$ and $]AB[$. F it is intersection of lines AD and CE. We denote as $\|CD\| = a, \|BD\| = b, \|DF\| = c$ and $\|AF\| = d$. Determine the ratio $\frac{\|BE\|}{\|AE\|}$ in terms of a, b, c and d https://cdn.artofproblemsolving.com/attachments/5/7/856c97045db2d9a26841ad00996a2b809adda.png