Flanders Math Olympiad 2012

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by parmenides51

1 Our class decides to have a alpha - beta - gamma tournament. This party game is always played in groups of three. Any possible combination of three players (three students or two students and the teacher) plays the game 1 time. The player who wins gets 1 point. The two losers get no points. At the end of the tournament, miraculously, all students have as many points. The teacher has 3 points. How many students are there in our class?

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2 Let n be a natural number. Call a the smallest natural number you need to subtract from n to get a perfect square. Call b the smallest natural number that you must add to n to get a perfect square. Prove that $n - ab$ is a perfect square.

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3 (a) Show that for any angle θ and for any natural number m:

$$|\sin m\theta| \le m|\sin\theta|$$

(b) Show that for all angles θ_1 and θ_2 and for all even natural numbers m:

$$|\sin m\theta_2 - \sin m\theta_1| \le m|\sin(\theta_2 - \theta_1)|$$

(c) Show that for every odd natural number m there are two angles, resp. θ_1 and θ_2, exist for which the inequality in (b) is not valid.

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4 In $\triangle ABC, \angle A = 66^o$ and $|AB| < |AC|$. The outer bisector in A intersects BC in D and $|BD| = |AB| + |AC|$. Determine the angles of $\triangle ABC$.
