

Flanders Math Olympiad 2013

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by parmenides51

- 1 A six-digit number is *balanced* when all digits are different from zero and the sum of the first three digits is equal to the sum of the last three digits. Prove that the sum of all six-digit balanced numbers is divisible by 13.

- 2 2013 smurfs are sitting at a large round table. Each of them has two tickets. on each card represents a number from $\{1, 2, \dots, 2013\}$ such that each of the numbers from this set occurs exactly twice. Every smurf takes the card every minute with the smaller of the two numbers, it smurfs on to its left neighbor and receives a card from his right neighbor. Show that there will come a time when a smurf has two cards with the same number.

- 3 Anton the ant takes a walk along the vertices of a cube. He starts at a vertex and stops when it reaches this point again. Between two vertices it moves over an edge, a side face diagonal or a space diagonal. During the rout it visits each of the other vertices exactly *once* and nowhere intersects its road already traveled.
 - (a) Show that Anton walks along at least one edge.
 - (b) Show that Anton walks along at least two edges.

- 4 Consider (in the plane) three concentric circles with radii 1, 2 and 3 and equilateral triangle Δ such that on each of the three circles is one vertex of Δ . Calculate the length of the side of Δ .
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