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- 1 A man in a rowing boat is at point A at a distance of 2 kilometers from a straight coastline. By first rowing in to a point P and then strolling along the coast he reaches point B, which is at a distance of 5 kilometers from C, which is the point on the coast closest to A. The man's speed at rest is 3 kilometers per hour and his strolling speed is 5 kilometers per hour. Decide where P should go be placed between C and B so that the man gets from A to B in the shortest possible time.

- 2 In a right-angled triangle,  $a$  and  $b$  denote the lengths of the two catheti. A circle with radius  $r$  has the center on the hypotenuse and touches both catheti. Show that  $\frac{1}{a} + \frac{1}{b} = \frac{1}{r}$ .

- 3 Let  $x$  and  $y$  be positive numbers with  $x + y = 1$ . Show that

$$\left(1 + \frac{1}{x}\right) \left(1 + \frac{1}{y}\right) \geq 9.$$

- 4 Let  $a, b$  and  $c$  denote the side lengths and  $m_a, m_b$  and  $m_c$  of the median's lengths in an arbitrary triangle. Show that

$$\frac{3}{4} < \frac{m_a + m_b + m_c}{a + b + c} < 1$$

Also show that there is no narrower range that for each triangle that contains the fraction

$$\frac{m_a + m_b + m_c}{a + b + c}$$

- 5 In a hat are 1992 notes with all numbers from 1 to 1992. At random way, two bills are drawn simultaneously from the hat; the difference between the numbers on the two notes are written on a new note, which is placed in the hat, while the two drawn notes thrown away. It continues in this way until there is only one note left in it the hat. Show that there is an even number on this slip of paper.