

**Dutch Mathematical Olympiad 1984**

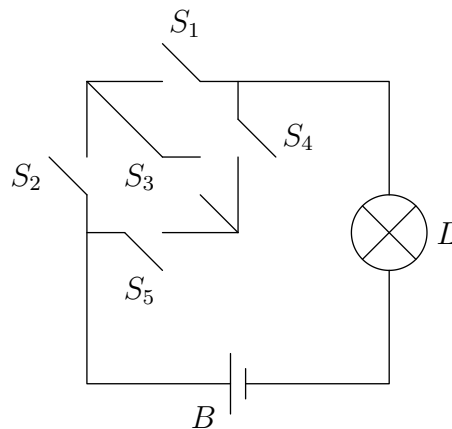
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by parmenides51

- 1 The circles  $C_1$  and  $C_2$  with radii  $r_1$  and  $r_2$  touch the line  $p$  at the point  $P$ .  $C_1$  lies entirely within  $C_2$ . Line  $q \perp p$  intersects  $p$  at  $S$  and touches  $C_1$  at  $R$ .  $q$  intersects  $C_2$  at  $M$  and  $N$ , where  $N$  is between  $R$  and  $S$ .
  - a) Prove that line  $PR$  bisects angle  $\angle MPN$ .
  - b) Calculate the ratio  $r_1 : r_2$  if line  $PN$  bisects angle  $\angle RPS$ .

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- 2 The circuit diagram drawn (see figure ) contains a battery  $B$ , a lamp  $L$  and five switches  $S_1$  to  $S_5$ . The probability that switch  $S_3$  is closed (makes contact) is  $\frac{2}{3}$ , for the other four switches that probability is  $\frac{1}{2}$  (the probabilities are mutually independent). Calculate the probability that the light is on.



- 3 For  $n = 1, 2, 3, \dots$   $a_n$  is defined by:

$$a_n = \frac{1 \cdot 4 \cdot 7 \cdot \dots \cdot (3n - 2)}{2 \cdot 5 \cdot 8 \cdot \dots \cdot (3n - 1)}$$

Prove that for every  $n$  holds that

$$\frac{1}{\sqrt{3n+1}} \leq a_n \leq \frac{1}{\sqrt[3]{3n+1}}$$

- 4 By placing parentheses in the expression  $1 : 2 : 3$  we can get two different number values:  $(1 : 2) : 3 = \frac{1}{6}$  and  $1 : (2 : 3) = \frac{3}{2}$ . Now brackets are placed in the expression  $1 : 2 : 3 : 4 : 5 : 6 : 7 : 8$ .

Multiple bracket pairs are allowed, whether or not in nest form.

- (a) What is the largest numerical value we can get, and what is the smallest?
  - (b) How many different number values can be obtained?
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