Dutch Mathematical Olympiad 1986

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1 $f(x)=\frac{12x+9}{19x+86},\ x\neq -\frac{86}{19}$

Prove that $\exists!\ x_o \in R\ \ \forall h_1,h_2 \in R [f(x_0+h_1)f(x_0-h_1)=f(x_0+h_2)f(x_0-h_2)]$, and calculate x_0.

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2 Prove that for all positive integers n holds that

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+...+\frac{1}{(2n-1)\cdot 2n}=\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{2n}$$

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3 The following apply: $a,b,c,d \ge 0$ and $abcd = 1$

Prove that

$$a^2+b^2+c^2+d^2+ab+ac+ad+bc+bd+cd \ge 10$$

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4 The lines a and b are parallel and the point A lies on a. One chooses one circle γ through A tangent to b at B. a intersects γ for the second time at T. The tangent line at T of γ is called t.

Prove that independently of the choice of γ, there is a fixed point P such that BT passes through P.

Prove that independently of the choice of γ, there is a fixed circle δ such that t is tangent to δ.
