**Dutch Mathematical Olympiad 1987**

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by parmenides51

**1** Solve into $N$:

$$a^2 = 2^b + c^4$$

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**2** For $x > 0$ , prove that

$$\frac{1}{2\sqrt{x+1}} < \sqrt{x+1} - \sqrt{x} < \frac{1}{2\sqrt{x}}$$

and for all $n \geq 2$ prove that

$$1 < 2\sqrt{n} - \sum_{k=1}^{n} \frac{1}{\sqrt{k}} < 2$$

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**3** There are two kinds of creatures living in the flatland of Pentagonia: the Spires ($S$) and the Bones ($B$). They all have the shape of an isosceles triangle: the Spiers have an apical angle of $36^o$ and the bones an apical angle of $108^o$.
Every year on *Great Day of Division* (September 11 - the day this Olympiad was held) they divide into pieces: each $S$ into two smaller $S$'s and a $B$; each $B$ in an $S$ and a $B$. Over the course of the year they then grow back to adult proportions. In the distant past, the population originated from one $B$-being. Deaths do not occur.
Investigate whether the ratio between the number of Spires and the number of Bones will eventually approach a limit value and if so, calculate that limit value.

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**4** On each side of a regular tetrahedron with edges of length $1$ one constructs exactly such a tetrahedron. This creates a dodecahedron with $8$ vertices and $18$ edges. We imagine that the dodecahedron is hollow. Calculate the length of the largest line segment that fits entirely within this dodecahedron.

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