Dutch Mathematical Olympiad 1988

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1 The real numbers $x_1, x_2, ..., x_n$ and $a_0, a_1, ..., a_{n-1}$ with $x_i \neq 0$ for $i \in \{1, 2, .., n\}$ are such that

$$(x - x_1)(x - x_2)...(x - x_n) = x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$$

Express $x_1^{-2} + x_2^{-2} + ... + x_n^{-2}$ in terms of $a_0, a_1, ..., a_{n-1}$.

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2 Given is a number a with $0 \leq \alpha \leq \pi$. A sequence $c_0, c_1, c_2, ...$ is defined as

$$c_0 = \cos \alpha$$

$$C_{n+1} = \sqrt{\frac{1 + c_n}{2}} \ for \ \ n = 0, 1, 2, ...$$

Calculate $\lim_{n\to\infty} 2^{2n+1}(1 - c_n)$

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3 For certain a, b, c holds: $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c}$
Prove that for all odd n holds,

$$\frac{1}{a^n} + \frac{1}{b^n} + \frac{1}{c^n} = \frac{1}{a^n + b^n + c^n}.$$

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4 Given is an isosceles triangle ABC with $AB = 2$ and $AC = BC = 3$. We consider squares where A, B and C lie on the sides of the square (so not on the extension of such a side). Determine the maximum and minimum value of the area of such a square. Justify the answer.
