Dutch Mathematical Olympiad 1989

www.artofproblemsolving.com/community/c3236160

by parmenides51

1 For a sequence of integers $a_1, a_2, a_3, ...$ with $0 < a_1 < a_2 < a_3 < ...$ applies:

$$a_n = 4a_{n-1} - a_{n-2} \;\; for \;\; n > 2$$

It is further given that $a_4 = 194$. Calculate a_5.

---

2 Given is a square $ABCD$ with $E \in BC$, arbitrarily. On CD lies the point F is such that $\angle EAF = 45^o$. Prove that EF is tangent to the circle with center A and radius AB.

---

3 Calculate

$$\sum_{n=1}^{1989} \frac{1}{\sqrt{n+\sqrt{n^2-1}}}$$

---

4 Given is a regular n-sided pyramid with top T and base $A_1A_2A_3...A_n$. The line perpendicular to the ground plane through a point B of the ground plane within $A_1A_2A_3...A_n$ intersects the plane TA_1A_2 at C_1, the plane TA_2A_3 at C_2, and so on, and finally the plane TA_nA_1 at C_n. Prove that $BC_1 + BC_2 + ... + BC_n$ is independent of choice of B's.

---

5 $\forall k \in N \;\; \exists n(k) \in N, a(k) : 0 < a(k) < 1[(1+\sqrt{2})^{2k+1} = n(k) + a(k)]$

Prove: $(n(k) + a(k))a(k) = 1$, for all $k \in N$, and calculate $\lim_{k\to\infty} a(k)$
