

German National Olympiad 2004www.artofproblemsolving.com/community/c3236485

by sqrtX, indybar

– Day 1

1 Find all real numbers x, y satisfying the following system of equations

$$\begin{aligned}x^4 + y^4 &= 17(x + y)^2 \\ xy &= 2(x + y).\end{aligned}$$

2 Let k be a circle with center M . There is another circle k_1 whose center M_1 lies on k , and we denote the line through M and M_1 by g . Let T be a point on k_1 and inside k . The tangent t to k_1 at T intersects k in two points A and B . Denote the tangents (different from t) to k_1 passing through A and B by a and b , respectively. Prove that the lines a, b , and g are either concurrent or parallel.

3 Prove that for every positive integer n there is an n -digit number z with none of its digits 0 and such that z is divisible by its sum of digits.

– Day 2

4 For a positive integer n , let a_n be the integer closest to \sqrt{n} . Compute

$$\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_{2004}}.$$

5 Prove that for four positive real numbers a, b, c, d the following inequality holds and find all equality cases:

$$a^3 + b^3 + c^3 + d^3 \geq a^2b + b^2c + c^2d + d^2a.$$

6 Is there a circle which passes through five points with integer co-ordinates?
