www.artofproblemsolving.com/community/c3236491
by parmenides51

1 This figure is cut out from a sheet of paper. Folding the sides upwards along the dashed lines, one gets a (non-equilateral) pyramid with a square base. Calculate the area of the base.
https://1.bp.blogspot.com/-lPpfHqfMMRY/XzcBIiF-n2I/AAAAAAAAMW8/nPs_mLe5C8srcxNz45Wg-_SqHlRAsAmigCLcBGAsYHQ/s0/2005%2BMohr%2Bp1.png

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2 Determine, for any positive real number a, the number of solutions (x, y) to the system of equations

$$\begin{cases} |x| + |y| = 1 \\ x^2 + y^2 = a \end{cases}$$

where x and y are real numbers.

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3 The point P lies inside $\triangle ABC$ so that $\triangle BPC$ is isosceles, and angle P is a right angle. Furthermore both $\triangle BAN$ and $\triangle CAM$ are isosceles with a right angle at A, and both are outside $\triangle ABC$. Show that $\triangle MNP$ is isosceles and right-angled.
https://1.bp.blogspot.com/-i9twOChu774/XzcBLP-RIXI/AAAAAAAAMXA/n5TJCOJypeMVW28-9GDG4st5C4s0/2005%2BMohr%2Bp3.png

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4 Fourteen students each write an integer number on the board. When they later meet their math teacher Homer Grog, they tell him that no matter what number they erased on the board, then the remaining numbers could be divided into three groups at once sum. They also tell him that the numbers on the board were integer numbers. Is it now possible for Homer Grog to determine what numbers the students wrote on the board?

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5 For what real numbers p has the system of equations

$$\begin{cases} x_1^4 + \dfrac{1}{x_1^2} = px_2 \\ x_2^4 + \dfrac{1}{x_2^2} = px_3 \\ ... \\ x_{2004}^4 + \dfrac{1}{x_{2004}^2} = px_{2005} \\ x_{2005}^4 + \dfrac{1}{x_{2005}^2} = px_1 \end{cases}$$

just one solution $(x_1, x_2, ..., x_{2005})$, where $x_1, x_2, ..., x_{2005}$ are real numbers?
