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by parmenides51

1   The star shown is symmetric with respect to each of the six diagonals shown. All segments connecting the points $A_1, A_2, ..., A_6$ with the centre of the star have the length 1, and all the angles at $B_1, B_2, ..., B_6$ indicated in the figure are right angles. Calculate the area of the star.
`https://1.bp.blogspot.com/-Rso2aWGUq_k/XzcAm4BkAvI/AAAAAAAAMW0/277afcqTfCgZOHshf_`
`6ce2XpinWWR4SZACLcBGAsYHQ/s0/2006%2BMohr%2Bp1.png`

2   Determine all sets of real numbers (x, y, z) which fulfills

$$\begin{cases} x + y = 2 \\ xy - z^2 = 1 \end{cases}$$

3   A natural number n, which is at most 500, has the property that when one chooses at at random among the numbers $1, 2, 3, ..., 499, 500$, then the probability is $\frac{1}{100}$ for m to add up to n. Determine the largest possible value of n.

4   Of the numbers $1, 2, 3, .., 2006$, ten different ones must be selected. Show that you can pick ten different numbers with a sum greater than 10039 in more ways than you can select ten different numbers with a sum less than 10030.

5   We consider an acute triangle ABC. The altitude from A is AD, the altitude from D in triangle ABD is DE, and the altitude from D in triangle ACD is DF.
a) Prove that the triangles ABC and AFE are similar.
b) Prove that the segment EF and the corresponding segments constructed from the vertices B and C all have the same length.