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- 1 Triangle ABC lies in a regular decagon as shown in the figure. What is the ratio of the area of the triangle to the area of the entire decagon? Write the answer as a fraction of integers.

https://1.bp.blogspot.com/-Ld_-4u-VQ5o/Xzb-KxPX0wI/AAAAAAAAAMWg/-qPtaI_04CQ3vvVc1wDTj3Soos0/2007%2BMohr%2Bp1.png

- 2 What is the last digit in the number 2007^{2007} ?

- 3 A cunning dragon guards a princess. To overcome the dragon and to win the princess you must solve the following task: The dragon has in some of the fields i the columned hall (see figure) with the numbers $1 - 8$. Even in the rest of the fields you can place numbers $9 - 36$. The numbers $1 - 36$ must be arranged so that any turn that starts with one enters from either the south or the west, and ends up going out towards either the north or east, goes through at least one number from the 5 table. (On the figure are north, south, east and west indicated by N, S, E and W). Georg wants to win the princess. Is it possible to be done?

<https://cdn.artofproblemsolving.com/attachments/0/7/2ad1b7f944847ee6d3c614ea6c2656865808.png>

- 4 The figure shows a 60° angle in which are placed 2007 numbered circles (only the first three are shown in the figure). The circles are numbered according to size. The circles are tangent to the sides of the angle and to each other as shown. Circle number one has radius 1. Determine the radius of circle number 2007.

https://1.bp.blogspot.com/-1bsLIXZpol4/Xzb-Nk6ospI/AAAAAAAAAMWk/jrx1zVYKbNELTW1DQ3zL9qc_22b2IJF6QCLcBGAsYHQ/s0/2007%2BMohr%2Bp4.png

- 5 The sequence of numbers a_0, a_1, a_2, \dots is determined by $a_0 = 0$, and

$$a_n = \begin{cases} 1 + a_{n-1} & \text{when } n \text{ is positive and odd} \\ 3a_{n/2} & \text{when } n \text{ is positive and even} \end{cases}$$

How many of these numbers are less than 2007?