AoPS Online
Art of Problem Solving

Final Round - Switzerland 2008

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–	Day 1
1	Let ABC be a triangle with $\angle BAC \neq 45^o$ and $\angle ABC \neq 135^o$. Let P be the point on the line AB with $\angle CPB = 45^o$. Let O_1 and O_2 be the centers of the circumcircles of the triangles ACP and BCP respectively. Show that the area of the square CO_1PO_2 is equal to the area of the triangle ABC.
2	Determine all functions $f : R^+ \to R^+$, so that for all $x, y > 0$: $$f(xy) \le \frac{xf(y) + yf(x)}{2}$$
3	Show that each number is of the form $$2^{5^{2^{5^{\cdots}}}} + 4^{5^{4^{5^{\cdots}}}}$$ is divisible by 2008, where the exponential towers can be any independent ones have height ≥ 3.
4	Consider three sides of an $n \times n \times n$ cube that meet at one of the corners of the cube. For which n is it possible to use this completely and without overlapping to cover strips of paper of size 3×1? The paper strips can also do this glued over the edges between these cube faces.
5	Let $ABCD$ be a square with side length 1. Find the locus of all points P with the property $AP \cdot CP + BP \cdot DP = 1$.
–	Day 2
6	Determine all odd natural numbers of the form $$\frac{p+q}{p-q},$$ where $p > q$ are prime numbers.
7	An 8×11 rectangle of unit squares somehow becomes disassembled into 21 contiguous parts . Prove that at least two of these parts, except for rotations and reflections have the same shape.

8 Let $ABCDEF$ be a convex hexagon inscribed in a circle . Prove that the diagonals AD, BE and CF intersect at one point if and only if

$$\frac{AB}{BC} \cdot \frac{CD}{DE} \cdot \frac{EF}{FA} = 1$$

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9 There are 7 lines in the plane. A point is called a *good* point if it is contained on at least three of these seven lines. What is the maximum number of *good* points?

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10 Find all pairs(a, b) of positive real numbers with the following properties:
(i) For all positive real numbers x, y, z, w holds $x + y^2 + z^3 + w^6 \geq a(xyzw)^b$.
(ii) There is a quadruple (x, y, z, w) of positive real numbers such that in equality (i) applies.
