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1 The figure shows a game board with 16 squares. At the start of the game, two cars are placed in different squares. Two players A and B alternately take turns, and A starts. In each turn, the player chooses one of the cars and moves it one or more squares to the right. The left-most car may never overtake or land on the same square as the right-most car. The first player which is unable to move loses.
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png
(a) Prove that A can win regardless of how B plays, if the two cars start as shown in the figure.
(b) Determine all starting positions in which B can win regardless of how A plays.

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2 The figure shows a rectangle, its circumscribed circle and four semicircles, which have the rectangle's sides as diameters. Prove that the combined area of the four dark gray crescentshaped regions is equal to the area of the light gray rectangle.
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3 A sequence $x_0, x_1, x_2, ...$ is given by $x_0 = 8$ and $x_{n+1} = \frac{1+x_n}{1-x_n}$ for $n = 0, 1, 2, ...$ Determine the number x_{2013}.

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4 The positive integer a is greater than 10, and all its digits are equal. Prove that a is not a perfect square.

(A perfect square is a number which can be expressed as n^2 , where n is an integer.)

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5 The angle bisector of A in triangle ABC intersects BC in the point D. The point E lies on the side AC, and the lines AD and BE intersect in the point F. Furthermore, $\frac{|AF|}{|FD|} = 3$ and $\frac{|BF|}{|FE|} = \frac{5}{3}$. Prove that $|AB| = |AC|$.
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f1f-JWwCLcBGAsYHQ/s0/2013%2BMohr%2Bp5.png
