Argentina National Olympiad 2010

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–	Level 3
–	Day 1
1	Given several integers, the allowed operation is to replace two of them by their non-negative difference. The operation is repeated until only one number remains. If the initial numbers are $1, 2, \ldots, 2010$, what can be the last remaining number?
2	Let ABC be a triangle with $\angle C = 90^o$ and $AC = 1$. The median AM intersects the incircle at the points P and Q, with P between A and Q, such that $AP = QM$. Find the length of PQ.
3	The positive integers a, b, c are less than 99 and satisfy $a^2 + b^2 = c^2 + 99^2$. . Find the minimum and maximum value of $a + b + c$.
–	Day 2
4	Find the sum of all products $a_1a_2...a_{50}$, where $a_1, a_2, ..., a_{50}$ are distinct positive integers, less than or equal to 101, and such that no two of them add up to 101.
5	21 numbers are written in a row. u, v, w are three consecutive numbers so $v = \frac{2uw}{u+w}$. The first number is $\frac{1}{100}$, the last one is $\frac{1}{101}$. Find the 15th number.
6	In a row the numbers $1, 2, ..., 2010$ have been written. Two players, taking turns, write $+$ or $\times$ between two consecutive numbers whenever possible. The first player wins if the algebraic sum obtained is divisible by 3; otherwise, the second player wins. Find a winning strategy for one of the players.