(Russian) Open Regional Olympiad in Geometry named after V.I. prof. S.A. Anishchenko for students in grades 8-11 (8-10 in 2007)

www.artofproblemsolving.com/community/c3268916

by parmenides51

VIII *I - one and only round*

p1. Three points are given in the plane of the drawing, which are the centers of the excircles of the triangle ABC. Construct triangle ABC. (A circle is called excircle if it touches one of the sides of the triangle and the extensions of the other two of its sides). It is enough to offer a reasonable plan for constructing a triangle.

p2. Given a circle with center at a point O and a point P lying inside the circle. Segment AB is given. Construct a chord of a circle passing through the point P and equal to the segment AB. Find out how many solutions the problem can have depending on the length of the segment AB and the position of the point P.

p3. In a pentagon $ABCDE$, diagonals AC and EC bisect angles $\angle A$ and $\angle E$, respectively. Find the area of the pentagon $ABCD$ if $\angle B = 55^o$, $\angle D = 125^o$ and the area of triangle ACE is 10.

p4. In an isosceles triangle ABC ($AB = BC$), $\angle A = 75^o$. The bisector of angle $\angle A$ intersects side BC at point K. Find the distance from point K to the base AC if $BK = 10$.

p5. CD is the angle bisector of triangle ABC, where $\angle C = 90^o$. Prove that $CD = \frac{AC\cdot BC\sqrt2}{AC+BC}$.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c3260315_`) and here (`https://artofproblemsolving.com/community/c3188020_`).

IX *I - one and only round*

p1. The area of a right triangle is equal to S. From the midpoint of the median drawn to the hypotenuse of this triangle, perpendiculars are drawn to its sides. Find the area of a triangle whose vertices are the feet of the perpendiculars.

p2. In the rhombus $ABCD$ with diagonals $AC = d_1$ and $BD = d_2$, heights CE and CK are drawn from the vertex C of an obtuse angle. Find the area of quadrilateral $AECK$.

p3. The hypotenuse of a right triangle is the side of the square outside the triangle. Find the distance between the vertex of the right angle of the triangle and the center of the square if the sum of the legs of the triangle is d.

p4. In an isosceles triangle ABC ($AB = BC$), altitude BD is drawn. From the point D, the perpendicular DE is drawn on the side AB. Prove that the line passing through the point B and the midpoint of the segment DE is perpendicular to the line EC.

p5. A convex quadrilateral $ABCD$ is given. The diagonals divide it into 4 triangles, the areas of which are equal to S_1, S_2, S_3 and S_4 in the order of going around its vertices. Prove that $S_1 \cdot S_3 = S_2 \cdot S_4$.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c3260315_) and here (https://artofproblemsolving.com/community/c3188020_).

X *I - one and only round*

p1. Given a circle and point Aon it. Find the locus of orthocenters of triangles ABC inscribed in this circle if $\angle A = 60^o$.

p2. Given a rectangle $ABCD$ and a point P. The lines passing through the points A and B and perpendicular to the lines PC and PD, respectively, intersect at the point K. Prove that the line PK is perpendicular to AB.

p3. In an isosceles triangle ABC, $AC = CB$, $\angle ACB = 100^o$. The point M inside the triangle is chosen so that $\angle MAB = 30^o$, $\angle MBA = 20^o$. Find the value of the angle $\angle MCA$.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c3260315_) and here (https://artofproblemsolving.com/community/c3188020_).

X 4 The edge of a regular tetrahedron is equal to a. Find the largest value of the area of the projection of this tetrahedron onto a plane.

$\frac{a^2}{2}$

X 5 A sphere inscribed in the tetrahedron $ABCD$ touches the face ABC at point M. Prove that the angle $\angle AMC$ is equal to half the sum of the angles of the three-dimensional quadrilateral $ABCD$.
