(Russian) Open Regional Olympiad in Geometry named after V.I. prof. S.A. Anishchenko for students in grades 8-11

www.artofproblemsolving.com/community/c3268941

by parmenides51

VIII *II - one and only round*

p1. A point lying inside a parallelogram is connected to all its vertices. Prove that the sums of the areas of the opposite triangles into which the parallelogram is divided are equal.

p2. In an isosceles right-angled triangle ABC, the bisector of an acute angle B is drawn, which intersects the leg AC at point L. Having the segments CL and LA as the sides, squares are constucted. Prove that the area of one square is twice the area of the other.

p3. A circle with center O of radius R intersects the sides of angle $\angle AOB$ at points A and B. A straight line passing through point A intersects circle (O,R) at point C, and the extewnsion of side OB of the given angle at point D such that point O lies between B and D. Prove that if $CD = R$, then $\angle AOB = 3\angle ADB$.

p4. $ABCD$ is rhombus . Triangles ADM and DCK are regular. Points M and B lie on the same side of AD, points K and B lie on opposite sides of CD. Prove that B, M, K lie on the same line.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c3260315_`) and here (`https://artofproblemsolving.com/community/c3188020_`).

IX *II - one and only round*

p1. Two circles touch externally at point A. BC is their common external tangent that intersects the line of centers at point K, points B and C are the touchpoints. The line ℓ is drawn through K and is perpendicular to BC. Lines AB and AC intersect ℓ at points P and E. Prove that $KP = KE$.

p2. In triangle ABC, $AC = 10$, $BC = 12$, $\angle CAB = 2\angle CBA$. Find the length of side AB.

p4. Point C is selected on segment BA. Three circles are constructed having segments AC, BC, BA as diameters. CT is perpendicular on AB. Circles of radii r_1 and r_2 are inscribed in the regions bounded by semicircles and the line segment CT. Prove that $r_1 = r_2$.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c3260315_`) and here (`https://artofproblemsolving.com/community/c3188020_`).

IX 3 In the triangle ABC, $AB = AC$, $\angle BAC = 80^o$. Point M is taken inside the triangle such that $\angle MBC = 30^o$, $\angle MCB = 10^o$. Find the value of the angle $\angle AMC$.

X *II - one and only round*

p1. In a right triangle ABC, the radius of the inscribed circle is r, AH is the altitude drawn from the vertex of the right angle. Circles are inscribed in each of the resulting right triangles. Find the distance between their centers.

p2. Point E is selected on the bisector of angle $\angle AOB$, EA and EB are perpendicular on the sides of the angle. A point P is chosen arbitrarily on the segment AB. The line MK is drawn through P, perpendicular on PE. Points M and K lie on the sides of the angle. Prove that P is the midpoint of MK.

p5. Two circles and a straight line are given. Construct a square so that two of its opposite vertices lie on the circles, and the other two lie on a straight line.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c3260315_`) and here (`https://artofproblemsolving.com/community/c3188020_`).

X 3 same as IX 3

X 4 All edges of the triangular prism $ABCA_1B_1C_1$ are equal to a and $\angle AA_1C_1 = \angle AA_1B_1 = 60^o$. Find the total surface area of the prism.

$a\left(1+\frac{3\sqrt{3}}{2}\right)$

XI 1 The base of the pyramid is a right triangle. The two side edges of the pyramid and the leg of the base enclosed between them are 10 cm, 17 cm, 21 cm, respectively. The dihedral angles at the base of the pyramid are equal. Find the volume and area of its surface if the base of the height is the interior point of the base.

$V = 140\sqrt{7}$ cm^3, $S = 490$ cm^2

XI 2 The edge of the cube $ABCDA_1B_1C_1D_1$ is equal to a. Find the distance between BD_1 and DC_1.

$\frac{a\sqrt{6}}{6}$

XI *II - one and only round*

p3. The altitude BD is drawn in the triangle ABC ($AC = BC$). A perpendicular is drawn from point D on side AB. Prove that the line passing through point B and the midpoint of segment DE is perpendicular to line EC.

p4. BH is the altitude of triangle BAC. Perpendiculars are drawn from the point H on the other two sides of the triangle and to the altitudes drawn from the vertices A and C. Prove that the four points , the feet of the perpendiculars from H lie on the same line.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c3260315_) and here (https://artofproblemsolving.com/community/c3188020_).