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–	Day 1
1	Find all prime numbers p, q, r, k such that $pq + qr + rp = 12k + 1$
2	Find all positive real numbers (x, y, z) such that: $$x = \frac{1}{y^2 + y - 1}$$ $$y = \frac{1}{z^2 + z - 1}$$ $$z = \frac{1}{x^2 + x - 1}$$
3	Let ABC be an acute triangle and Γ its circumcircle. The lines tangent to Γ through B and C meet at P. Let M be a point on the arc AC that does not contain B such that $M \neq A$ and $M \neq C$, and K be the point where the lines BC and AM meet. Let R be the point symmetrical to P with respect to the line AM and Q the point of intersection of lines RA and PM. Let J be the midpoint of BC and L be the intersection point of the line PJ and the line through A parallel to PR. Prove that L, J, A, Q, and K all lie on a circle.
–	Day 2
4	Determine the maximum number of bishops that we can place in a 8×8 chessboard such that there are not two bishops in the same cell, and each bishop is threatened by at most one bishop. Note: A bishop threatens another one, if both are placed in different cells, in the same diagonal. A board has as diagonals the 2 main diagonals and the ones parallel to those ones.
5	The circumferences C_1 and C_2 cut each other at different points A and K. The common tangent to C_1 and C_2 nearer to K touches C_1 at B and C_2 at C. Let P be the foot of the perpendicular from B to AC, and let Q be the foot of the perpendicular from C to AB. If E and F are the symmetric points of K with respect to the lines PQ and BC, respectively, prove that A, E and F are collinear.
6	Let k be a positive integer and $a_1, a_2, \cdots, a_k$ digits. Prove that there exists a positive integer n such that the last $2k$ digits of 2^n are, in the following order, $a_1, a_2, \cdots, a_k, b_1, b_2, \cdots, b_k$, for certain digits $b_1, b_2, \cdots, b_k$

