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by randomusername

1 In fencing, you win a round if you are the first to reach 15 points. Suppose that when A plays against B, at any point during the round, A scores the next point with probability p and B scores the next point with probability $q = 1 - p$. (However, they never can both score a point at the same time.)

Suppose that in this round, A already has $14 - k$ points, and B has $14 - \ell$ (where $0 \leq k, \ell \leq 14$). By how much will the probability that A wins the round increase if A scores the next point?

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2 Consider a triangle ABC and a point D on its side $\overline{AB}$. Let I be a point inside $\triangle ABC$ on the angle bisector of ACB. The second intersections of lines AI and CI with circle ACD are P and Q, respectively. Similarly, the second intersection of lines BI and CI with circle BCD are R and S, respectively. Show that if $P \neq Q$ and $R \neq S$, then lines AB, PQ and RS pass through a point or are parallel.

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3 Let $Q = \{0,1\}^n$, and let A be a subset of Q with 2^{n-1} elements. Prove that there are at least 2^{n-1} pairs $(a,b) \in A \times (Q \setminus A)$ for which sequences a and b differ in only one term.
