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by randomusername

- 1 Let  $1 \leq k \leq n$  be integers. At most how many  $k$ -element subsets can we select from  $\{1, 2, \dots, n\}$  such that for any two selected subsets, one of the subsets consists of the  $k$  smallest elements of their union?

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- 2 Prove that for any finite set  $A$  of positive integers, there exists a subset  $B$  of  $A$  satisfying the following conditions:
  - if  $b_1, b_2 \in B$  are distinct, then neither  $b_1$  and  $b_2$  nor  $b_1 + 1$  and  $b_2 + 1$  are multiples of each other, and
  - for any  $a \in A$ , we can find a  $b \in B$  such that  $a$  divides  $b$  or  $b + 1$  divides  $a + 1$ .

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- 3 If  $p, q \in \mathbb{R}[x]$  satisfy  $p(p(x)) = q(x)^2$ , does it follow that  $p(x) = r(x)^2$  for some  $r \in \mathbb{R}[x]$ ?

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