

Peru IMO TST 2006

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by carlosbr

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Saturday, may 20.

Question 01

Find all (x, y, z) positive integers, such that:

$$\sqrt{\frac{2006}{x+y}} + \sqrt{\frac{2006}{y+z}} + \sqrt{\frac{2006}{z+x}},$$

is an integer.

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Spanish version (<http://www.mathlinks.ro/Forum/viewtopic.php?t=88509>)

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Question 02

Find all pairs (a, b) real positive numbers a and b such that:

$$[a[bn]] = n - 1,$$

for all n positive integer.

Note: $[x]$ denotes the integer part of x .

Spanish version (<http://www.mathlinks.ro/Forum/viewtopic.php?t=88510>)

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Question 03

In each square of a board drawn into squares of 2^n rows and n columns ($n \geq 1$) are written a 1 or a -1, in such a way that the rows of the board constitute all the possible sequences of length n that they are possible to be formed with numbers 1 and -1.

Next, some of the numbers are replaced by zeros.
 Prove that it is possible to choose some of the rows of the board
 (It could be a row only) so that in the chosen rows, is fulfilled that the
 sum of the numbers in each column is zero.

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Spanish version (<http://www.mathlinks.ro/Forum/viewtopic.php?t=88511>)
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Question 04

In an acute-angled triangle ABC draw up: its circumcircle w
 with center O , the circumcircle w_1 of the triangle AOC and
 the diameter OQ of w_1 . The points are chosen M and N on
 the straight lines AQ and AC , respectively, in such a way that
 the quadrilateral $AMBN$ is a parallelogram.

Prove that the intersection point of the straight lines MN and BQ belongs to the
 circumference w_1 .

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Spanish version (<http://www.mathlinks.ro/Forum/viewtopic.php?t=88513>)
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