Peru IMO TST 2007

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–	Grade level 1
1	Let P be an interior point of the semicircle whose diameter is AB ($\angle APB$ is obtuse). The incircle of $\triangle ABP$ touches AP and BP at M and N respectively. The line MN intersects the semicircle in X and Y. Prove that $\widehat{XY} = \angle APB$.
2	Let a, b, c be positive real numbers, such that: $a+b+c \geq \frac{1}{a}+\frac{1}{b}+\frac{1}{c}$. Prove that: $$a+b+c \geq \frac{3}{a+b+c}+\frac{2}{abc}.$$
3	Let N be a natural number which can be expressed in the form $a^2+b^2+c^2$, where a, b, c are integers divisible by 3. Prove that N can be expressed in the form $x^2+y^2+z^2$, where x, y, z are integers and any of them are divisible by 3.
4	Let be a board with 2007×2007 cells, we colour with black P cells such that: • there are no 3 colored cells that form a L-trinomes in any of its 4 orientations Find the minimum value of P, such that when you colour one cell more, this configuration can't keep the condition above.
–	Grade level 2
1	Let k be a positive number and P a Polynomio with integer coeficients. Prove that exists a n positive integer such that: $P(1)+P(2)+\cdots+P(N)$ is divisible by k.
2	Let ABC be a triangle such that $CA \neq CB$, the points A_1 and B_1 are tangency points for the ex-circles relative to sides CB and CA, respectively, and I the incircle. The line CI intersects the cincumcircle of the triangle ABC in the point P. The line that trough P that is perpendicular to CP, intersects the line AB in Q. Prove that the lines QI and A_1B_1 are parallels.
3	Let T a set with 2007 points on the plane, without any 3 collinear points. Let P any point which belongs to T.

Prove that the number of triangles that contains the point P inside and its vertices are from T, is even.

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4 Let a, b and c be sides of a triangle. Prove that: $\frac{\sqrt{b+c-a}}{\sqrt{b}+\sqrt{c}-\sqrt{a}}+\frac{\sqrt{c+a-b}}{\sqrt{c}+\sqrt{a}-\sqrt{b}}+\frac{\sqrt{a+b-c}}{\sqrt{a}+\sqrt{b}-\sqrt{c}} \leq 3$
