National Math Olympiad (3rd Round) 1993

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1 Prove that there exist infinitely many positive integers which can't be represented as sum of less than 10 odd positive integers' perfect squares.

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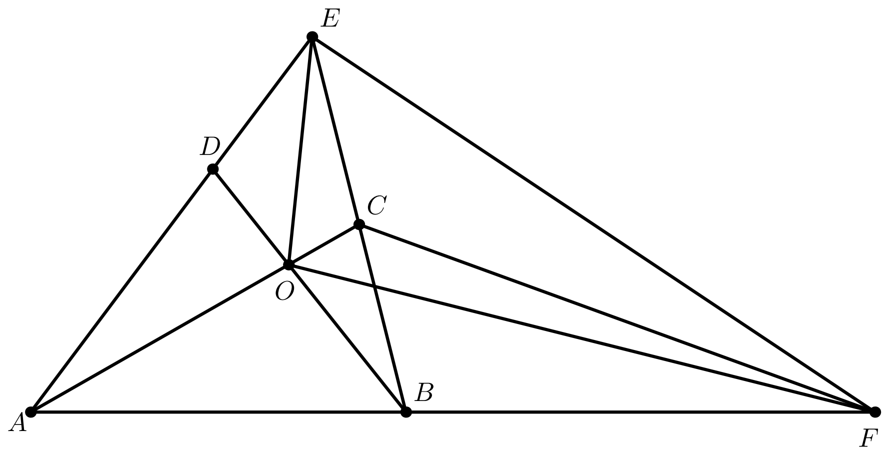
2 In the figure below, area of triangles $AOD, DOC,$ and AOB is given. Find the area of triangle OEF in terms of area of these three triangles.



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4 Prove that there exists a subset S of positive integers such that we can represent each positive integer as difference of two elements of S in exactly one way.

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5 In a convex quadrilateral $ABCD$, diagonals AC and BD are equal. We construct four equilateral triangles with centers O_1, O_2, O_3, O_4 on the sides sides AB, BC, CD, DA outside of this quadrilateral, respectively. Show that $O_1O_3 \perp O_2O_4$.

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6 Let $x_1, x_2, \ldots, x_{12}$ be twelve real numbers such that for each $1 \leq i \leq 12$, we have $|x_i| \geq 1$. Let $I = [a, b]$ be an interval such that $b - a \leq 2$. Prove that number of the numbers of the form $t = \sum_{i=1}^{12} r_i x_i$, where $r_i = \pm 1$, which lie inside the interval I, is less than 1000.
