National Math Olympiad (3rd Round) 2001

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1   Find all functions $f : \mathbb{Q} \longrightarrow \mathbb{Q}$ such that: $f(x) + f(\frac{1}{x}) = 1 \ 2f(f(x)) = f(2x)$

2   Does there exist a sequence $\{b_i\}_{i=1}^{\infty}$ of positive real numbers such that for each natural m:

$$b_m + b_{2m} + b_{3m} + \cdots = \frac{1}{m}$$