Hungary-Israel Binational 1990

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by M4RI0, April

1 Prove that there are no positive integers x and y such that $x^2 + y + 2$ and $y^2 + 4x$ are perfect squares

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2 Let ABC be a triangle where $\angle ACB = 90^\circ$. Let D be the midpoint of BC and let E, and F be points on AC such that $CF = FE = EA$. The altitude from C to the hypotenuse AB is CG, and the circumcentre of triangle AEG is H. Prove that the triangles ABC and HDF are similar.

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3 Prove that:

$$\frac{1989}{2} - \frac{1988}{3} + \frac{1987}{4} - \cdots - \frac{2}{1989} + \frac{1}{1990} = \frac{1}{996} + \frac{3}{997} + \frac{5}{998} + \cdots + \frac{1989}{1990}$$

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4 A rectangular sheet of paper with integer length sides is given. The sheet is marked with unit squares. Arrows are drawn at each lattice point on the sheet in a way that each arrow is parallel to one of its sides, and the arrows at the boundary of the paper do not point outwards. Prove that there exists at least one pair of neighboring lattice points (horizontally, vertically or diagonally) such that the arrows drawn at these points are in opposite directions.
