Hungary-Israel Binational 1991

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by M4RI0, April

1 Suppose $f(x)$ is a polynomial with integer coefficients such that $f(0) = 11$ and $f(x_1) = f(x_2) = ... = f(x_n) = 2002$ for some distinct integers $x_1, x_2, ..., x_n$. Find the largest possible value of n.

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2 The vertices of a square sheet of paper are A, B, C, D. The sheet is folded in a way that the point D is mapped to the point D' on the side BC. Let A' be the image of A after the folding, and let E be the intersection point of AB and $A'D'$. Let r be the inradius of the triangle EBD'. Prove that $r = A'E$.

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3 Let $\mathcal{H}_n$ be the set of all numbers of the form $2 \pm \sqrt{2 \pm \sqrt{2 \pm \ldots \pm \sqrt{2}}}$ where "root signs" appear n times.
(a) Prove that all the elements of $\mathcal{H}_n$ are real.
(b) Computer the product of the elements of $\mathcal{H}_n$.
(c) The elements of $\mathcal{H}_{11}$ are arranged in a row, and are sorted by size in an ascending order. Find the position in that row, of the elements of $\mathcal{H}_{11}$ that corresponds to the following combination of $\pm$ signs:

$$+ + + + + - + + - + -$$

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4 Find all the real values of λ for which the system of equations $x+y+z+v = 0$ and $(xy + yz + zv) + \lambda (xz + xv + yv) = 0$, has a unique real solution.
