Hungary-Israel Binational 1994

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by April

1 Let m and n be two distinct positive integers. Prove that there exists a real number x such that $\frac{1}{3} \le \{xn\} \le \frac{2}{3}$ and $\frac{1}{3} \le \{xm\} \le \frac{2}{3}$. Here, for any real number y, $\{y\}$ denotes the fractional part of y. For example $\{3.1415\} = 0.1415$.

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2 Let $a_1, \ldots, a_k, a_{k+1}, \ldots, a_n$ be n positive numbers ($k < n$). Suppose that the values of $a_{k+1}, a_{k+2}, \ldots, a_n$ are fixed. Choose the values of $a_1, a_2, \ldots, a_k$ that minimize the sum $\sum_{i,j,i \neq j} \frac{a_i}{a_j}$

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3 Three given circles have the same radius and pass through a common point P. Their other points of pairwise intersections are A, B, C. We define triangle $A'B'C'$, each of whose sides is tangent to two of the three circles. The three circles are contained in $\triangle A'B'C'$. Prove that the area of $\triangle A'B'C'$ is at least nine times the area of $\triangle ABC$

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4 An [i]$n-m$ society[/i] is a group of n girls and m boys. Prove that there exists numbers n_0 and m_0 such that every [i]$n_0 - m_0$ society[/i] contains a subgroup of five boys and five girls with the following property: either all of the boys know all of the girls or none of the boys knows none of the girls.
